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MODELING OF PARAFFIN DEPOSITION IN FLOW OF A GAS-OIL MIXTURE IN TUBES

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The paper presents a scheme of the formation and growth of paraffin deposits on internal walls of the hoisting column of a well with allowance for diffusive transport of the particles of a solid phase to the wall of the hoisting column, for temperature and concentration conditions for these particles (that ensure the formation of a solid phase), and for thermal effects of the paraffin formation.

The production from oil and gas-oil wells of West Siberian fields is frequently complicated by solid deposits on internal walls of the hoisting column that are generally of two types. The first of them is gas hydrates. A prerequisite for hydrate formation is the presence of water (or its vapor) and light hydrocarbon components which, while dissolving in water at certain temperatures and pressures, form a solid phase [1, 2]. The second type of deposits usually observed in exploiting oil and gas-oil wells represents high-molecular hydrocarbon systems hereafter called paraffin deposits.

At least two factors are needed for their formation in a well:

1) the gas-liquid mixture should contain heavy (high-molecular) components which are immediate predecessors of the paraffin deposits forming on the internal walls of the hoisting column;

2) the temperature mode in a well (especially near its walls) should admit of the existence of a solid phase, specifically, the presence, near the well walls, of such zones where the temperature becomes lower than or equal to the crystallization temperature of a heavy component.

Study [3] revealed that the most probable mechanism of paraffin formation is a crystallization mechanism, i.e., under conditions when the gas-liquid flow is capable of transporting heavy components, the formation and subsequent accretion of paraffin deposits occur due to crystal growth directly on the surface of the underground equipment. The buildup of a solid phase on the well walls involves diffusive and thermodiffusive transfer of heavy hydrocarbon components to the solid surface. The rate of formation of paraffin deposits depends also on the flow velocity or the well output. Therefore, for calculating a running position of the "gas-oil flow–paraffin" interface it is necessary to write the following relation:

$$\rho_s^0 \frac{\partial \delta_s}{\partial t} = - j_s - j_s' (w) .$$

An expression for the rate of entrainment of paraffin deposits by the gas-liquid flow $j_s'(w)$ is taken in the form

$$j_s' (w) = \gamma w \, |w| ,$$

where γ is the proportionality factor whose prediction requires experimental or practical data. However, there are no such data in literary or other sources known to the authors. In this connection, we assume that the formation of a solid phase is determined mainly by the first term ($j_s'(w) = 0$).

For the rate of formation of paraffin deposits we write

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$$j_s = \frac{\overset{0}{\rho}_{\text{liq}} D\,\text{Sh}}{2R}\,(k_{\text{liq(s)}} - k_{\text{liq(s)e}})\,, \tag{1}$$

where $\text{Sh} = \sqrt{2R\nu_{\text{liq}}/D}$ is the Sherwood number.

The dimensionless mass transfer coefficient Sh in the adopted form suggests that mass transfer proceeds only in a thin diffusive boundary layer of the thickness $\delta^{(D)}$ ($\delta^{(D)} << R$) near the "solid–liquid" interface. A characteristic value of $\delta^{(D)}$ can be estimated from the relation

$$\delta^{(D)} \cong \sqrt{Dt_{\text{liq,w}}}\,,$$

where $t_{\text{liq,w}}$ is a certain characteristic time of contact of a liquid particle with the channel wall, $t_{\text{liq,w}} \cong 2R/\nu_{\text{liq}}$.

On the basis of the Schröder equation [4] for $k_{\text{liq(s)e}}$ as a function of T_σ it is possible to use the expression

$$k_{\text{liq(s)e}} = k^*_{\text{liq(s)e}} \exp\left(-T^*_{\text{s}}/T_\sigma\right).$$

Expression (1) has been derived assuming that the rate of paraffin formation is totally limited by the diffusion mechanism of transfer of heavy hydrocarbon components in a liquid. However, generally speaking, this quantity is also dependent on the rate of particle attachment to the solid surface [5]. In view of this circumstance, expression (1) can be generalized in the form

$$j_s = \frac{\overset{0}{\rho}_{\text{liq}}}{2R}\,(k_{\text{liq(s)}} - k_{\text{liq(s)e}})\left(\frac{1}{D\,\text{Sh}} + \frac{1}{D\,\text{Sh}^*}\right)^{-1},$$

where the coefficient $D\,\text{Sh}^*$ defines the rate of particle attachment to the surface. In the case where the condition $\text{Sh} >> \text{Sh}^*$ is fulfilled (which is characteristic of oil paraffins [3]), the rate of formation of a solid phase is limited by the diffusion mechanism of transfer of its predecessor to the interface and the thickness of paraffin deposits can be predicted from equation (1).

However, determining the thickness of paraffin deposits from relation (1) appears to be difficult, since the mean mass concentration of a heavy component $k_{\text{liq(s)}}$ should be known. In practice it is more convenient to obtain the rate of formation of a solid phase knowing the temperatures of flow, of the walls of the well tubes, etc., since more or less tried methods are available for their prediction.

In connection with the foregoing, we rearrange expression (1). First we assume that the formation of a solid phase is determined mainly by a loss of the liquid flowability near the walls because the liquid congeals with a decrease in the temperature. This assumption implies that the liquid contains plenty of heavy components generating a solid phase, and the rate of paraffin deposition is determined from the condition of heat balance on the surface of solid deposits

$$Lj_s = -q_{\text{w}} - \lambda_{\text{s}}\left(\frac{\partial T_{\text{s}}}{\partial r}\right)_R, \quad \frac{\partial \delta_{\text{s}}}{\partial t} = \frac{j_s}{\overset{0}{\rho}_{\text{s}}}. \tag{2}$$

Besides, we assume that the inner surface of a solid phase is isothermal and its temperature is equal to the crystallization temperature of a heavy component T_{e} in a liquid phase of the hydrocarbon mixture flowing in a well. Accordingly, we can write $T_\sigma = T_{\text{e}}$.

The rate of heat transfer between the surface of a solid phase and the flow is taken in the form

$$q_{\text{w}} = \beta_{\text{w}}(T - T_\sigma)\,. \tag{3}$$

The heat transfer coefficient β_{w} depends on the structure of the gas-oil flow in a well and on specific features of the flow in the surface layer near a solid phase or channel walls.

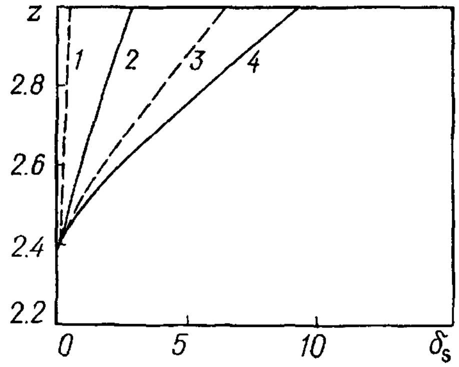

Fig. 1. Profiles of paraffin deposits on internal walls of the hoisting column of a well at different instants of time: 1) $t = 4$ days, 2) 8, 3) 10, 4) 11. z, km; δ_s, mm.

Since the characteristic time of complete closing of the flow area of the hoisting column of a well is much longer than the characteristic time of transition of the temperature profile inside a solid phase $t_s = \delta_s^2/\chi_s$ to a steady state, we can assume that the temperature distribution in a solid layer T_s at each instant of time satisfies the equation

$$r^{-1}\frac{\partial}{\partial r}\left(\chi_s r\frac{\partial T_s}{\partial r}\right) = 0 \quad (R < r < R_0). \tag{4}$$

It should be noted that adopting the equation of heat conduction in the form (4) implies also that heat transfer along a well by a solid phase is disregarded. Based on equation (4) we obtain the following relation for the temperature distribution in a solid layer:

$$T_s = T_0 + \frac{T_\sigma - T_0}{\ln (R/R_0)}\ln (r/R_0). \tag{5}$$

With account for expressions (3) and (5), equation (2) can be brought to the form

$$\rho_s^0 L\frac{\partial \delta_s}{\partial t} = \beta_s (T_0 - T_\sigma) - \beta_w (T - T_\sigma) \quad \left(\beta_s = \frac{\lambda_s}{R \ln (R/R_0)}\right). \tag{6}$$

The presented scheme (6) for describing the paraffin deposition suggests that its rate is completely determined from the condition of heat balance. Besides, the surface of a solid phase is assumed to be isothermal. We now extend scheme (6) to the case when the deposition rate is a function of mass transfer of a heavy component to the surface of a solid phase.

Suppose that there is no depletion of a heavy component ($k_{\text{liq(s)}} = \text{const}$) and the surface temperature of the solid phase in the deposition region varies slightly. Then, the T_σ dependence of $k_{\text{liq(s)}}$ can be taken in a linear approximation. Expression (1) for the rate of formation of a solid phase is rewritten as

$$\rho_s^0\frac{\partial \delta_s}{\partial t} = K(T_e - T_\sigma) \quad \left(K = \frac{\rho_{\text{liq}}^0 D\,\text{Sh}}{2R}\left(\frac{\partial k_{\text{liq(s)e}}}{\partial T_\sigma}\right)_{T_e}\right). \tag{7}$$

Eliminating T_σ from relations (6) and (7) results in

$$\frac{\partial \delta_s}{\partial t} = \frac{\beta_s (T_0 - T_e) - \beta_w (T - T_e)}{\rho_s^0 [L + (\beta_w - \beta_s)/K]}. \tag{8}$$

In this case, the surface temperature of the solid phase is obtained from the expression

$$T_\sigma = \frac{\beta_w T + K L T_e - \beta_s T_0}{\beta_w + K L - \beta_s}.$$

Using scheme (8) we calculated the evolution of paraffin deposits in time (see the figure). The following values were employed in calculations: $R_0 = 0.0365$ m, $\rho_s^0 = 900$ kg/m^3, $\lambda_s = 0.27$ W/(m·K), $L = 2 \cdot 10^5$ m^2/sec^2, and $D = 10^{-9}$ m^2/sec. It was also supposed that the crystallization temperature of paraffin is 30°C, there is a liquid (oil) in the intertubular space of the well, and the temperature on the outer boundary of the well is equal to geothermal. In calculations paraffin was assumed to deposit in those sections of the well where the temperature of the internal wall of the hoisting column becomes lower than or equal to the temperature of the onset of paraffin depostiion. In order to exclude uncertainty in the expression for β_s , δ_s at the initial instant of time was taken to be equal to 0.1 mm.

The obtained profiles for the form and rate of growth are in agreement with practical data.

Thus, the proposed mathematical model allows a determination of the thickness of oil paraffin deposits on internal walls of the hoisting column at different instants of time from the start of the well operation.

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NOTATION

T, temperature of a gas-oil flow; T_0, T_σ, and T_e, temperature of the internal wall of a hoisting column, on the surface of a solid phase, and of the onset of crystallization of a heavy component, respectively; t, time; r, radius; q_w and β_w, rate and coefficient of heat transfer between a gas-oil flow and a solid phase or the wall of a hoisting column, respectively; R_0, inside radius of a hoisting column; δ_s, thickness of paraffin deposits ($R = R_0 - \delta_s$); λ_s and χ_s, thermal conductivity and thermal diffusivity of paraffin; j_s, rate of formation of paraffin deposits related to heat and mass transfer of heavy components to a solid surface; $j_s'(w)$, rate of paraffin entrainment by a gas-oil flow; w, mean flow rate of a gas-oil mixture; ρ_s^0 and ρ_{liq}^0, density of solid and liquid phases; D, diffusion coefficient; $k_{liq(s)}$ and $k_{liq(s)e}$, mean mass concentration of a heavy component in the liquid and its equilibrium concentration at a temperature equal to the surface temperature of a solid phase T_σ, respectively; v_{liq}, linear velocity of a liquid phase; $k_{liq(s)e}^*$ and T_s^*, empirical approximation parameters; L, specific heat of paraffin formation. Subscripts: s and liq, parameters of the solid and liquid phases, respectively; (s), parameters of the solid component; w, parameters on the channel wall; e, equilibrium parameters; (D), quantities related to diffusion; 0, parameters of the initial state.

REFERENCES

1. E. A. Bondarev, L. N. Gabysheva, and M. A. Kanibolotskii, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 105-112 (1982).
2. E. D. Sloan, *Clathrate Hydrates of Natural Gases*, New York (1990).
3. V. P. Tronov, *Mechanism of the Formation of Tar-Paraffin Deposits and Their Control* [in Russian], Moscow (1969).
4. V. A. Kirillin, A. E. Sheindlin, and É. É. Shpil'rain, *Thermodynamics of Solutions* [in Russian], Moscow (1980).
5. R. F. Strickland-Constable, *Kinetics and Mechanism of Crystallization* [Russian translation], Leningrad (1971).